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OUTILS ET MÉTHODES INTELLIGENTES POUR LES RADIO-SCIENCES

Modèle proxy pour la quantification d'incertitudes dans un système de mesure quasi-optique en bande J

A surrogate model for uncertainty quantification in a J-band quasi-optical measurement system

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Abstract

Quasi-optical systems are widely used nowadays to measure the dielectric permittivity of materials under test (MUT) on a large frequency band. Indeed, they allow non-destructive measurements of the latter as well as permittivity extraction of non-solid material using a tank. For example, Figure 1 shows the system used at IMT Atlantique *[1]*. The objective is to measure with high accuracy the S-parameters of the material and then to retrieve its permittivity by solving an inverse problem.

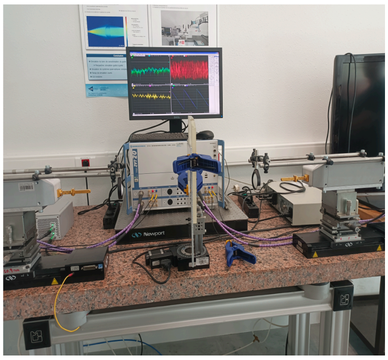

Figure 1: Quasi-optic system.

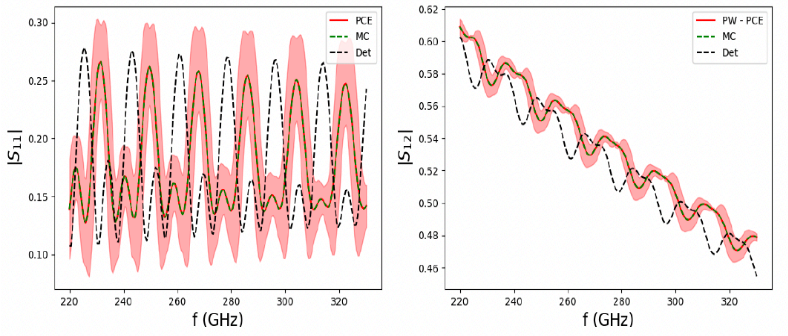

Figure 2: Mean and confidence interval computed with the PCE proxy. Comparison with Monte Carlo.

In this context, the forward model must be accurate, and it is important to characterize its variations with different parameters, such as the positions of the lens, the angular tilt of the MUT, or the thickness of any air gap for a multilayer material. This task corresponds to uncertainty quantification. A straightforward way to obtain statistical information from uncertain inputs is to use the Monte Carlo method. Since it is a non-intrusive method, it does not require modifying the simulation code. Indeed, we draw $\boldsymbol{N_s}$ samples of the inputs and calculate the associated outputs. From this, one can easily estimate the mean and other statistical quantities. Nonetheless, even if from the law of large number, we are sure of its convergence, the number of samples it requires is quite large and the overall computation time drastically increases. It shall be noted that accurate forward models, such as those in **[1, 2]**, can be computationally expensive. In this work, to overcome this limitation, we propose to use a surrogate model based on polynomial chaos expansion **[3]**. We denote by $\boldsymbol{X}$ the set of uncertain inputs of size M, e.g. the tilt, the thicknesses, etc., and assume that all its components are independent. The core idea is to decompose the random vector in an orthogonal basis of polynomial adapted to each joint law as $\boldsymbol{X}(\boldsymbol{\theta}) = \sum \boldsymbol{X}^{\boldsymbol{i}}\boldsymbol{\Psi}^{\boldsymbol{i}}(\boldsymbol{\theta})$ where $\boldsymbol{\Psi}_{\boldsymbol{i}}$ correspond to the set of multivariate polynomials chosen to form an orthogonal basis for the underlying distribution of $\boldsymbol{X}$, and $\boldsymbol{X}^{\boldsymbol{i}}$ the projection of the latter on each polynomial. These projections can be analytically calculated for usual distributions. From Céa's lemma we know that the output variable, here each S-parameter $\boldsymbol{S}_{\boldsymbol{ij}} \in \backslash mathbb\{C\}$, can be decomposed onto the same set of multivariate polynomials, where the decomposition coefficient are unknowns. Our goal is thus to calculate them efficiently to obtain the associate surrogate model that allows for fast computations of any statistical information **[3]**. Here, we use a projection based non-intrusive approach **[4]**, based on Gauss quadrature, where we can use directly the forward method as a black box. Note that since the S-parameters are complex we also use a quadrature to compute the first and second order moment of their magnitudes or phases. To validate our approach, we consider a three-layer material consisting of the first MUT, an air gap, and the second MUT and use a simple plane-wave approximation **[5]** to calculate the associated S-parameters. In this setting, the air gap thickness, the MUT thicknesses and the tilt of the MUT are assumed to be random variables. The first two variables are considered to follow log-normals law since they cannot be negative while the last one is supposed to be Gaussian. In Figure 2, we show the mean of the magnitudes of the S-parameters computed with the proposed surrogate (red line) or the Monte Carlo method (green dashed line). The phases have also been calculated but not shown here. For the latter we used 100000 samples. We also plot the associated 90% confidence interval. Finally, as a comparison we show the deterministic case (black dashed line), where the mean of each variable has been used for the simulation. Our first conclusion is that the proposed surrogate model performs well: its results align with Monte Carlo simulations while requiring 60 times less computational time. In addition, we observe that uncertainty quantification methods are necessary in this context, since it results that a deterministic simulation does not capture the inherent variations and is not accurate, as shown in Figure 2.

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