

Évaluation de l'Exposition Électromagnétique au Rapport Signal Interférence en Milieu Urbain

Evaluation of Electromagnetic Exposure to Signal-to-Interference Ratio in Downlink Cellular Networks

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Abstract

This study develops a stochastic geometry framework for analyzing Electromagnetic Field (EMF) exposure and the Signal-to-Interference Ratio (SIR) in the downlink scenario in urban environments. Signal-to-Interference Ratio (SIR) reflects communication quality and link reliability, while EMF exposure measures absorbed electromagnetic energy with respect to regulatory safety limits. These two dimensions are usually analyzed separately, leaving the trade-off between communication performance and human safety insufficiently addressed. A new parameter exposure-to-SIR (ESR) ratio is introduced to demonstrate the ratio between human exposure and signal quality. In addition, the cumulative distribution function (CDF) of ESR is derived for a general case and a special case with numerical validation is also presented. The framework enables the evaluation of EMF exposure and signal quality trade-off, quantifying the probability that a user achieves a target SIR while keeping EMF exposure below a safety limit. The result also shows an agreement in Monte Carlo simulation and the analytical framework. The rightward shift of the ESR CDF indicates that, in a probabilistic sense, users are more likely to experience higher exposure relative to the achieved SIR.

1 Introduction

The evolution of wireless communication systems towards 5G-Advanced and 6G is characterized by network densification, especially in urban environments. While denser base stations (BSs) in these new technologies promise continuous connectivity and superior data rates, they simultaneously intensify the complexity of the electromagnetic environment. Humans are directly immersed in this environment, raising concerns about exposure to electromagnetic fields. This creates a challenge in ensuring high Quality of Service (QoS) while strictly limiting human exposure to Electromagnetic Fields (EMF). Signal-to-Interference Ratio (SIR) serves as the metric for performance optimization and link reliability, whereas EMF exposure assessment relies on electromagnetic field or power density against regulatory thresholds. As these two aspects are usually treated separately, the balance between communication performance and human safety remains limited. To address this, the study proposes a parameter that quantifies the ratio between EMF exposure and SIR to capture the trade-off between human exposure and signal quality for sustainable network development in urban areas. Stochastic geometry (SG) provides a mathematical framework to model dense BSs and users as random point process, such as Poisson point process (PPP). It was used to evaluate coverage probability a decade ago [1], and it continues to prove its effectiveness by providing a trackable model for EMF exposure evaluation in both uplink and downlink scenarios [2], as well as the joint metrics of EMF exposure and coverage in cellular networks [3]. This paper introduces the Exposure-to-SIR (ESR) that enables the evaluation of EMF exposure and SIR through its cumulative distribution function (CDF) in a general case and provides analytical expression for a selected special case.

2 System Model

The distribution of base stations are modeled follow a homogeneous Poisson Point Process (PPP) denoted by ϕ , with density λ . Under homogeneous PPP model, the probability density function (PDF) of the distance r between the typical user and its serving BS (the BS has a smallest distance to the user) is $f_r(R) = 2\pi\lambda R e^{-\pi\lambda R^2}$ [1]. This approach allows for a tractable analysis of the aggregate interference and exposure levels across the network area, and a downlink scenario is considered.

A path loss model is denoted as $l(r) = r^{-\alpha}$ is used with path loss exponent $\alpha > 2$ and Rayleigh fading, where the channel gain follows $h \sim \text{Exp}(\mu)$. The network is assumed to be interference-limited, and thus thermal noise is ignored. The SIR

at the tagged user, therefore, is formulated as $SIR = \frac{S}{I} = \frac{P_t G h l(R_0)}{\sum_{i \in \phi \setminus \{r_0\}} P_t G k_i l(R_i)}$, where R_0 is the distance from tagged user to its nearest BS which is taken as the serving BS, and R_i denotes the distance to the i -th interfering base stations. The EMF exposure is defined by the total received power density as follows $\mathcal{E} = S + I = P_t G h l(R_0) + \sum_{i \in \phi \setminus \{r_0\}} P_t G k_i l(R_i)$. The parameter that quantifies the ratio between EMF exposure and SIR – Exposure-to-SIR (ESR) is expressed

$$ESR = \frac{\mathcal{E}}{SIR} = \frac{S + I}{S/I} = I + \frac{I^2}{S} \quad (1)$$

Leveraging stochastic geometry, the cumulative distribution function (CDF) of ESR is derived as follows

$$\Pr[ESR < y] = \int_{r>0} \left(2\pi\lambda r e^{-\pi\lambda r^2} \int_0^y \exp\left(-\frac{i^2 a(r)}{y-i}\right) f_i(i) di \right) dr \quad (2)$$

where $f_i(i)$ is PDF of interference I and can be obtained by using the Gil-Pelaez theorem.

This analytical framework enables the evaluation of the trade-off between human exposure and signal quality by quantifying the probability that a user achieves a target SIR threshold while keeping EMF exposure below a prescribed safety limit. It therefore provides a unified parameter to assess how the network should be deployed to maintain performance and keep exposure below the safety limit in urban environments.

3 Numerical Result

Considering the special case in which the path loss exponent $\alpha = 4$. The channel gains of both the serving and interfering links are assumed to follow exponential distribution with unit means, i.e., $Exp(1)$. The CDF of ESR in this case is given by

$$\Pr[ESR < y] = \int_{r>0} \left(2\pi\lambda r e^{-\pi\lambda r^2} \int_0^y \left(\exp\left(-\frac{i^2 a(r)}{y-i}\right) \left(\frac{1}{\pi} \int_0^\infty \operatorname{Re} \left[e^{-jt} \times \exp\left(-\pi\lambda \sqrt{-jt P_t G} \arctan\left(\frac{1}{r^2} \sqrt{-jt P_t G}\right)\right] \right) dt \right) \right) di \right) dr \quad (3)$$

The result of this special case is then validated through both Monte Carlo simulations and the analytical expression. The Monte Carlo simulations are performed within a disk of radius $R_m = 5000$ (in normalized length units). The result is illustrated in the figure below

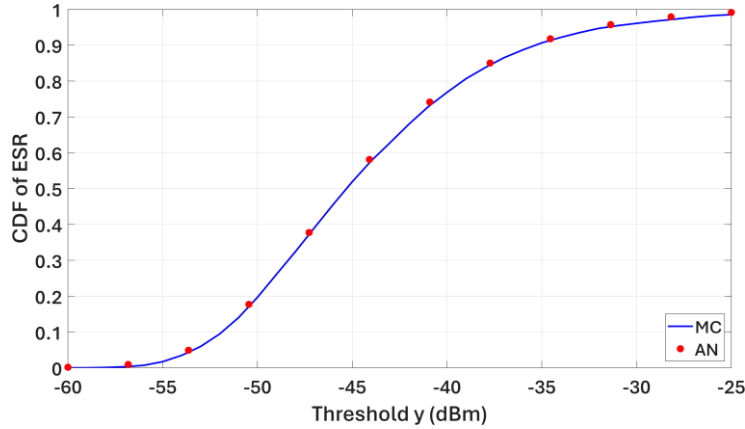


Figure 1. CDF of ESR

In Figure 1, the CDF of ESR obtained from Monte Carlo simulation and the analytical expression for the special case are shown. A good agreement is observed between analytical results and Monte Carlo line over the range of thresholds, which validates the correctness of the derived expression. The trade-off between EMF exposure and signal quality follows a distribution that can be characterized by the proposed analytical framework. As the ESR threshold increases, the CDF also increases, meaning that a larger fraction of users experiences an exposure-to-SIR ratio below this threshold. This indicates that most users in a simulation area have the exposure level remains moderate relative to the achieved signal quality.

Figure 2 demonstrates the CDF of ESR for different BSs densities λ with a path loss exponent $\alpha = 4$ under the path loss model $l(r) = r^{-\alpha}$. The results are illustrated by Monte Carlo simulations and the analytical framework results. As the increment of BSs, the ESR CDR tends to move towards higher threshold values, i.e., denser BSs deployments improve signal availability while simultaneously increasing the higher EMF exposure relative to the achieved SIR. Consequently, the CDF enables a probabilistic evaluation of the behavior of EMF exposure and SIR, reflecting a trade-off between improved signal availability and increased EMF exposure relative to the achieved SIR.

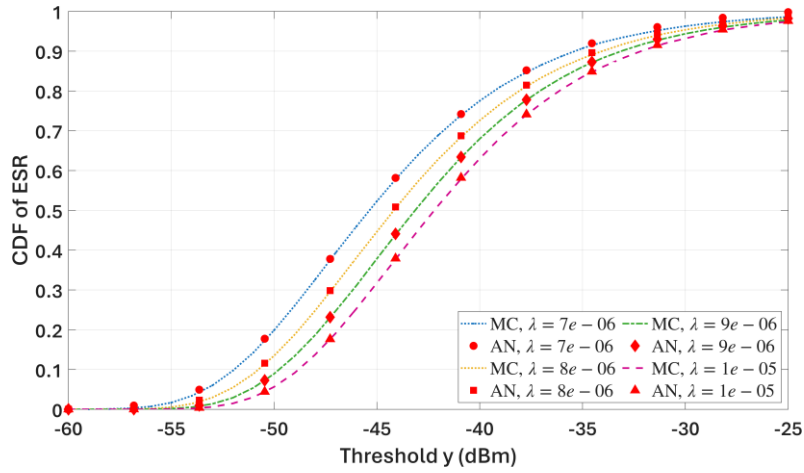


Figure 2. CDF of ESR with different BSs densities

4 Conclusion

The paper provides a stochastic geometry framework to evaluate the trade-off between electromagnetic exposure and SIR in the downlink of cellular networks. A unified metric exposure-to-SIR (ESR) is introduced and an integral form of ESR cumulative distribution function is derived, enabling a probabilistic characterization of exposure relative to the achieved SIR. The verification of the analytical results and Monte Carlo simulation shown an agreement, the results also demonstrate the shift to the right of ESR when density of BS increase indicating that users are more likely to experience higher exposure relative to the achieved SIR. These findings suggest that network design should not rely solely on coverage/SIR metrics but also account for exposure-related constraints. Future work will extend the model to further assess the ESR behavior in more realistic settings.

References

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